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De-singularization by rotation

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Abstract: We consider certain BPS supergravity solutions of string theory which have singularities and we show that the singularity goes away when we add angular momentum. These smooth solutions enable us to obtain *global* AdS_3 as the near horizon geometry of a BPS brane system in an asymptotically flat space.

Keywords: D-branes, Black Holes in String Theory, AdS-CFT and dS-CFT Correspondence.

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Contents

1. Introduction

It is well known that some BPS states in four and five dimensional supergravity theories can be realized as non-singular extremal black holes with non-zero horizon area. This is the situation for generic black hole charges. However, there are some cases where the area of the horizon becomes zero and the geometry becomes singular. For example, this happens for 1/4 BPS states of string theory on T^5. In this paper we show that by considering 1/4 BPS states with maximal angular momentum we can produce a completely non-singular geometry once we suitably include one of the internal dimensions. We were led to this solution by thinking about supersymmetric conical singularities in AdS_3. So first we analyze various aspects of supersymmetric AdS_3 spaces and conical singularities [1]. When we are dealing with AdS_3 we can consider the theory with NS-NS or RR boundary conditions on the spatial circle. It known that the $M = 0$ BTZ black hole is a RR ground state [2]. We show that by introducing Wilson lines for U(1) gauge fields in AdS_3 we can also interpret other conical singularities as RR ground states. Even pure AdS_3 with a suitable Wilson line can be interpreted as a RR ground state. All these ground states are different in their U(1) charges.

If we view global AdS_3 as the near horizon region of a six dimensional rotating black string of string theory on $R^{1,4} \times S^1 \times M^4$ coming from D1-D5 branes wrapped[1] on S^1, then we can match the smooth *global* AdS_3 solution to asymptotically flat space in such a way that it preserves supersymmetry. In other words, by adding angular momentum we can find a smooth supergravity solution that corresponds to the D1-D5 system. These are solutions which have maximal angular momenta $J_L = \pm J_R = Q_1 Q_5/2 \equiv k/2$.[2]

[1] The D5 branes also wrap M^4.

[2] One can also view the system in the S-dual picture, involving $F1 - NS5$. Conformal models describing the $F1 - NS5$ system with couplings representing the angular momenta have been discussed in [3].

These AdS_3 geometries with Wilson lines can also have the interpretation of "giant gravitons" in AdS_3.

The proper interpretation of these solutions will involve a precise statement and understanding of the possible boundary conditions for the gauge fields that live on AdS_3. So in section 2 we review some facts about gauge fields and Chern Simons theory. In section 3 we describe the interpretation of the solutions from the AdS/CFT point of view. In section 4 we match the AdS_3 solutions to the asymptotically flat region. In section 5 we briefly remark about the Interpretation of these configurations as giant gravitons.

As this paper was in preparation we received [4] which has a great deal of overlap with this paper.

2. Some facts about Wilson lines and Chern Simons theory

Let us start by describing some facts about U(1) gauge fields. Suppose we have a plane described by coordinates ρ, φ, $ds^2 = d\rho^2 + \rho^2 d\varphi$. Then consider a gauge field with the connection $A_\varphi = a$ where a is any constant. We see that $F = 0$ everywhere in the plane except at the origin where it is a delta function. This is of course the familiar gauge field of a Bohm-Aharonov vortex. The interaction with the gauge field is normalized so that we get the phase $e^{i\int A}$ for the field with the minimal quantum of charge. We see that if a is an integer particles do not feel any field and indeed we can set A to zero by a gauge transformations $A \to A + d\epsilon$ where $\epsilon(\varphi + 2\pi) = \epsilon(\varphi) + 2\pi n$, with n integer. We need to specify the boundary conditions for the charged fields when we go around the origin. We will work with fixed boundary conditions for the fields and we will vary a. Suppose we have a fermionic field and we impose the boundary condition that ψ is periodic as it goes around the circle. Then if we set $a = 1/2$, the field will effectively become antiperiodic. This implies that the fermionic field will be totally continuous at the origin, since the minus sign is what we expect for a rotation by 2π.

Now let us suppose that we have Chern Simons theory on a solid cylinder $D_2 \times R$, where D_2 is a disk. Then we need to impose some boundary conditions on the gauge field. As shown in [5, 6] we can impose the boundary condition only on one component of A along the boundary. One way to understand this is to view the direction orthogonal to the boundary as time so that one realizes that the two components of A along the boundary are canonically conjugate variables. We will be interested in setting boundary conditions of the form $2A_- = A_0 - A_\varphi = 0$. It is easy to see that these boundary conditions are consistent. We choose these boundary conditions because it was shown in [7] that they are appropriate for gauge fields in AdS supergravities. Once we give these boundary conditions we can have a variety of states in the theory with various values of A_+ on the boundary. These values are $2A_+ = q/2k$, with q integer [6], and k the level of the Chern Simons theory. These various states can arise by inserting various Wilson lines in the interior. States with $q \to q \pm 2k$ are related by a large gauge transformation which does not vanish at the boundary. These transformations map physical states to other physical states in the boundary theory. From the point of view of the topological theory in the bulk, states

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with q and $q \pm 2k$ are equivalent. The U(1) charge of the state has the value of $\frac{1}{2\pi}\int A$ along the spatial circle. If we have a Wilson line of charge q in the interior, this value is $A_\varphi = q/(2k)$.

Similar remarks about CS theory apply when the gauge group is non-compact, such as $\mathrm{SL}(2,\mathbb{R})$. In this case we consider again configurations with vanishing field strength and with the same asymptotic boundary conditions. This implies that the space is locally AdS but not globally. For example, we can consider the conical space

$$ds^2 = -(r^2+\gamma^2)dt^2 + r^2 d\varphi^2 + \frac{dr^2}{r^2+\gamma^2}\,. \tag{2.1}$$

Locally this is an AdS_3 space, but at $r = 0$ we have a conical singularity if $\gamma \neq 1$.

3. Conical singularities and AdS/CFT

In this section we will apply some of the above remarks to supergravity theories on AdS_3. What we will describe is mainly contained in [1, 8]. We will consider supergravity theories with extra U(1) gauge fields on AdS_3. One example we have in mind is the case of string theory on $AdS_3 \times S^3 \times K3$, but other examples could be treated in a similar way. We will consider gravity theories on AdS_3 with at least $(2,2)$ supersymmetry. This implies that we will have $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ gauge fields. Pure three dimensional gravity on AdS_3 is given by an $\mathrm{SL}(2,\mathbb{R})^2$ Chern Simons theory, which we will use to describe the conical spaces. In this situation we could consider solutions with arbitrary Wilson lines for the $\mathrm{U}(1)_{L,R}$ gauge field as well as the $\mathrm{SL}(2,\mathbb{R})_{L,R}$ gauge fields. In principle these solutions are singular in the interior and we should not consider them, unless we have a good reason to think that the singularity will be resolved in the full theory.

In this paper we will consider singularities which preserve at least (2,2) supersymmetry. We will impose RR boundary conditions on the fields and we consider arbitrary Wilson lines. In order for the solution to be supersymmetric the Wilson line in the $\mathrm{SL}(2,\mathbb{R})$ part and the U(1) part should be essentially the same, we will later make this statement more precise. The boundary of AdS_3 is $R \times S^1$. We normalize charges so that a fermion carries integer charge under $\mathrm{U}(1)_{R,L}$. As standard in AdS/CFT, the boundary conditions on all supergravity fields correspond to the microscopic definition of the "lagrangian" of the CFT, including the periodicities of the fields as we go around the circle, etc. We can then consider all solutions to the supergravity equations with given boundary conditions. Different solutions correspond to different states in the boundary CFT. Now let us choose RR boundary conditions for the CFT and sugra fields on the spatial boundary circle. We will impose the boundary condition $A^L_- = A^R_+ = 0$ for $\mathrm{U}(1)_{L,R}$.[3]

We will consider flat gauge fields with $\mathrm{U}(1)_{R,L}$ connections given by constant values $A^L_+ = a_+$, $A^R_- = a_-$. Supersymmetry determines the three dimensional geometry. We consider spinors generating supersymmetry that are periodic when we go around the circle,

[3]Actually one could impose the boundary condition $A^L_- = \epsilon_L$, $A^R_+ = \epsilon_R$, where $\epsilon_{L,R}$ are some constants. These would correspond to left and right spectral flows with the parameters $\epsilon_{L,R}$.

since we said we are interested in the RR sector. The solution is then:[4]

$$
\begin{aligned}
ds_3^2 = & -\left[(r-\frac{a_+^2-a_-^2}{r})^2+4a_+^2\right]dt^2+\frac{dr^2}{(r-\frac{a_+^2-a_-^2}{r})^2+4a_+^2}+ \\
& +r^2\left(d\varphi+\frac{a_+^2-a_-^2}{r^2}dt\right)^2 \\
A_+^L = & \, a_+\,, \qquad A_-^R=a_-\,, \qquad A_-^L=A_+^R=0\,.
\end{aligned} \tag{3.1}
$$

In the particular case of $a_+ = a_- = \gamma/2$ the solution is

$$
\begin{aligned}
ds_3^2 &= -(r^2+\gamma^2)dt^2+r^2d\varphi^2+\frac{dr^2}{r^2+\gamma^2} \\
A_+^L &= A_-^R=\frac{\gamma}{2}\,, \qquad A_-^L=A_+^R=0\,.
\end{aligned} \tag{3.2}
$$

All these configurations have zero energy, as implied by the RR sector super-algebra. The AdS_3 space in (3.2) seems to have negative energy, but one should add to this the energy that comes from the Wilson line. This additional energy comes from the "singleton" that lives at the boundary of AdS which encodes this degree of freedom. This was explicitly shown in [7]. So we have $L_0 = \bar{L}_0 = 0$. The angular momenta are half the U(1) charges, $J_L = J_R = k\gamma/2$. So we see that γ should be quantized as $\gamma = n/k$. We get zero energy states with various amounts of angular momenta.

So what is the interpretation of these spaces? which ones are allowed and which ones are not?. All these are supersymmetric solutions. Almost all of them are singular. Only if $\gamma = 1$ we see from (3.2) that we get a nonsingular solution. Let us discuss this solution first. The three dimensional geometry is that of AdS_3. The Wilson line around the origin of AdS_3 is such that it effectively changes the periodicity of fermionic fields from periodic to anti-periodic, so that they are smooth at the origin. This solution has angular momenta $J_L = J_R = k/2$. What is this state in the boundary CFT?. We know that the boundary CFT has a large number of RR vacua [9]. These vacua have angular momenta $|J_{L,R}| \leq k/2$. We see that the non-singular solution corresponds to a state with the maximal value of the angular momentum. From general arguments [10] we know that there is a single RR state with maximal value of the RR charge, it is the state that maps to the NS vacuum under spectral flow. Here we indeed see that the state we find is essentially the same as global AdS_3 which was identified as the NS vacuum. The only difference is that the Wilson lines imply that particle energies are shifted as they are shifted under spectral flow. Now we turn to the solutions with $\gamma \neq 1$. All those solutions contain a singularity at the origin. It is clear that starting from the solution with $\gamma = 1$ we can add supergravity particles that decrease the angular momentum and leave $L_0 = \bar{L}_0 = 0$, these particles, are of course, the chiral primaries discussed in [11], see also [12]. If we have particles with high values of the angular momentum, $l \gg 1$, l/k fixed, they will appear like very massive particles from the AdS_3 point of view and will give rise to the conical spaces with $\gamma < 1$. It is not possible to get the conical spaces

[4] We use conventions where $R_{AdS} = 1$.

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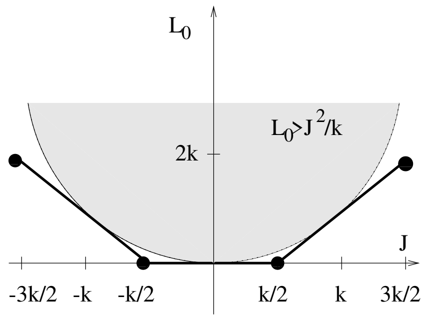

Figure 1: Spectrum of the theory in the RR sector. RR ground states have spins $|J| \leq k/2$. Quantum numbers that lie within the shaded region, with $L_0 > J^2/k$ can be carried by black holes. We have a similar figure for $\bar{L}_0$ and $\bar{J}$.

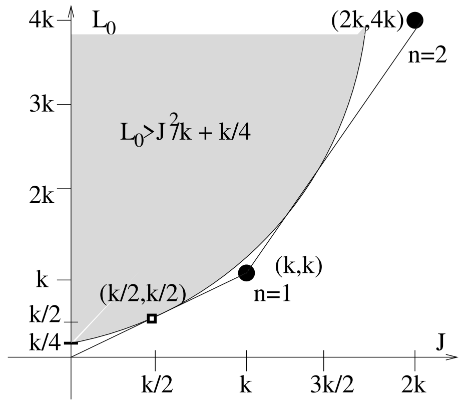

Figure 2: Spectrum of the theory in the NS sector sector. Quantum numbers that lie within the shaded region, with $L_0 > J^2/k + k/4$ can be carried by black holes. States with $J = nk$, $L_0 = n^2 k$ are AdS_3 spaces with Wilson lines.

with $\gamma > 1$ in this fashion, since all those supergravity particles would increase the energy and will remove us from the RR vacuum. In other words, by adding supergravity particles to the state with $J_L = J_R = k/2$ we can decrease the angular momentum while preserving the zero energy condition. If we try to increase J we would increase the energy.

We could imagine decreasing J by adding supergravity particles with low values of the spin, those gravity particles have wavefunctions which are quite extended in AdS. If we added them in a coherent state, we should be able to find classical solutions which are also smooth and do not have these conical singularities. Finding these solutions would require us to use the full six dimensional gravity equations. In other words, the fact that for $J < k/2$ we only found singular solutions does not mean that there are no non-singular solutions. A trivial example is the following. Consider the $AdS_3 \times S^3$ case. Now we have $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ symmetry groups. Let us pick the U(1)'s in the above discussion to be in the direction $\hat{3}$. Take the solution with maximal angular momentum and perform an $\mathrm{SU}(2)_{L,R}$ rotation in the $\hat{1}$ axis so that now the angular momentum points in the $\hat{2}$ direction. We get exactly the same AdS_3 space but now with a Wilson line $A_+^{L,2} = A_-^{R,2} = 1/2$ and the rest zero. This is a solution with zero U(1) charges but with no singularity, as opposed to the solution in (3.2) with $\gamma = 0$. Of course here we are treating these Wilson lines in a classical fashion. This is correct in the large k limit where we deal with macroscopic amounts of angular momentum.

It is easy to see that any solution which is AdS_3 and a Wilson line of the form $A_+^L = 1/2 + n$, $A_-^R = 1/2 + n'$ with integer n, n' will be non-singular. These solutions correspond to the spectral flow of the state with $n = n' = 0$. These solutions do not preserve the supersymmetries that the RR ground state preserve, but they do preserve other supersymmetries. These are the configurations that are related by spectral flow to the NS sector ground state.

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4. Non-singular solutions in asymptotically flat space

In this section we point out that the conical spaces, including the non-singular AdS_3 space with a Wilson line, can be extended to supersymmetric solutions of six dimensional supergravity that are asymptotic to $R^5 \times S^1$, with periodic boundary conditions on S^1. In other words, they represent BPS solutions in this six dimensional string theory.

We can find the solution by starting with the most general five dimensional black hole solution written in [13], lifting it up to six dimensions as in [14] and taking the extremal limit with zero momentum charge while keeping the angular momenta nonzero. The solution we obtain is parametrized by two angular momentum parameters which we take as $\gamma_{1,2}$: $J_{L,R} = \frac{k}{2}(\gamma_1 \mp \gamma_2)$ and can be written in the form:[5]

$$\begin{aligned}
\frac{ds_6^2}{\sqrt{k}} &= \frac{1}{h}(-dt^2 + d\varphi^2) + hf\left(d\theta^2 + \frac{r^2 dr^2}{(r^2+\gamma_1^2)(r^2+\gamma_2^2)}\right) - \\
&\quad - \frac{2}{hf}[(\gamma_2 dt + \gamma_1 d\varphi)\cos^2\theta d\psi + (\gamma_1 dt + \gamma_2 d\varphi)\sin^2\theta d\phi] + \\
&\quad + \left[h(r^2+\gamma_2^2) + (\gamma_1^2 - \gamma_2^2)\frac{\cos^2\theta}{hf}\right]\cos^2\theta d\psi^2 + \\
&\quad + \left[h(r^2+\gamma_1^2) - (\gamma_1^2 - \gamma_2^2)\frac{\sin^2\theta}{hf}\right]\sin^2\theta d\phi^2
\end{aligned} \tag{4.1}$$

where:

$$\begin{aligned}
f &= f(r,\theta) \equiv r^2 + \gamma_1^2\cos^2\theta + \gamma_2^2\sin^2\theta \\
h &= h(r,\theta) \equiv \frac{\sqrt{k}}{R_y^2}\left(1 + \frac{R_y^2 Q_1}{kf}\right)^{1/2}\left(1 + \frac{R_y^2 Q_5}{kf}\right)^{1/2}
\end{aligned} \tag{4.2}$$

and R_y is the radius of the S^1 parametrized by φ.

Setting the two angular momenta equal ($\gamma_2 = 0$, $\gamma \equiv \gamma_1$): $J_L = J_R = k\gamma/2$, we get the solution:

$$\begin{aligned}
\frac{ds_6^2}{\sqrt{k}} &= -\frac{1}{h}\left(dt + \frac{\gamma\sin^2\theta}{r^2+\gamma^2\cos^2\theta}d\phi\right)^2 + \frac{1}{h}\left(d\varphi - \frac{\gamma\cos^2\theta}{r^2+\gamma^2\cos^2\theta}d\psi\right)^2 + \\
&\quad + h\frac{r^2+\gamma^2\cos^2\theta}{r^2+\gamma^2}dr^2 + \\
&\quad + h\left[(r^2+\gamma^2\cos^2\theta)d\theta^2 + (r^2+\gamma^2)\sin^2\theta d\phi^2 + r^2\cos^2\theta d\psi^2\right]
\end{aligned} \tag{4.3}$$

In the decoupling near-horizon limit the metric reduces to a locally $AdS_3 \times S^3$, where the S^3 angles are defined as $\tilde\psi = \psi - \gamma\varphi$, $\tilde\phi = \phi - \gamma t$ [14].

---

[5] To relate our parameters and coordinates to the ones in eq. (4) of Cvetic and Larsen [14], we have $\gamma_{1,2} = \frac{R_y}{\sqrt{k}}(\cosh\delta_0\ell_{1,2} - \sinh\delta_0\ell_{2,1})$, $k = \lambda^4$ and $r = \frac{\sqrt{k}}{R_y}r^{C.L.}$, $t = \frac{1}{R_y}t^{C.L.}$, $\varphi = \frac{1}{R_y}y^{C.L.}$

Since the original angles are identified as: $\varphi \sim \varphi + 2\pi$, $\theta \sim \theta + \pi/2$, $\psi \sim \psi + 2\pi$, $\phi \sim \phi + 2\pi$, these new coordinates have the global identifications:

$$
\begin{aligned}
(\varphi, \tilde{\psi}) &\sim (\varphi, \tilde{\psi}) + 2\pi(1, -\gamma) \sim (\varphi, \tilde{\psi}) + 2\pi(0, 1) \\
\theta &\sim \theta + \frac{\pi}{2} \\
\tilde{\phi} &\sim \tilde{\phi} + 2\pi\,.
\end{aligned} \tag{4.4}
$$

For general (noninteger) values of the parameter γ the periodicities of the AdS_3 and the S^3 parts are still coupled, and the geometry obtained is singular.

The most interesting solution is the one with angular momenta $J_L = J_R = k/2$, when $\gamma = 1$, since it is non-singular. It is a non-singular, geodesically complete geometry. In its decoupling near-horizon limit, the space is globally a direct product $AdS_3 \times S^3$, as can be seen looking at the periodicities of the angles in (4.4).

It seems that the fact that this solution is non-singular is related to the fact that there are very few states in the CFT with similar values of the angular momenta.

It would be interesting to see if other BPS states in the AdS_3 region could be matched to the asymptotically flat region. Natural candidates are states with $L_0 = 0$, $J^R = k/2$ and $J_L = k/2 + nk$. In the near horizon limit these states have $A_-^R = 1/2$, $A_+^L = 1/2 + n$. The elliptic genus formula tells us that there is a single BPS state with these values of the angular momenta[6] (it is just the left spectral flow of the state we found above). If we tried to take a limit of the solutions in [13, 14], we would find that $J_R = 0$. It could be that we need to make a more general ansatz.

5. Super giant gravitons

In this section we consider NS-NS boundary conditions on the circle at the boundary. The ground state is AdS_3 with no Wilson lines for the U(1) gauge fields. We can consider the spectrum of chiral primaries, i.e. states with $L_0 = J_L$, $\bar{L}_0 = J_R$ as in [11]. From the CFT point of view we can calculate how many of these states we expect. It turns out that there is a single state with $J_{L,R} = 0$ and the number of states increases as we increase the values of $J_{L,R}$, it reaches a maximum at $J_L = J_R = k/2$ and then it starts decreasing again so that for $J_L = J_R = k$ we find just a single state again. In other AdS compactifications there is a maximum value for the single particle BPS states. In [16] it was shown that these states are realized as expanded branes, see also [17]. In $AdS_5 \times S^5$ the cutoff appears at $J = N$ [9], where J is the angular momentum on S^5. In AdS_3 the situation is different [11], there is an absolute cutoff on J at $J_L = J_R = k$, there are no chiral primary states beyond this value of J. By using the previous ideas about Wilson lines it is easy to see that this state is just AdS_3 with $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ Wilson lines equal to $A_+^L = A_-^R = 1$. We could roughly think about it as an AdS space which is just rotating as a whole. Only the "singleton" field is excited. The singleton is the mode that appears at the boundary from the Chern Simons theory in the interior. We can say that gravitons became so big that they live at the boundary of AdS.

[6] In principle, we also need to add the center of mass motion of the string in the transverse four dimensions [15].

So in the $AdS_3 \times S^3$ case the graviton with maximum angular momentum is not an expanded brane but just a different classical solution. This is in agreement with the fact that the maximal spin, k, is of the order of the inverse six dimensional Newton's constant, while in the $AdS_5 \times S^5$ this maximal value, N, is proportional to the square root of Newton's constant. Notice that objects such as long strings [18, 19] are not of concern here since we can work at a point in moduli space where there is no finite energy long string at infinity. This is possible if Q_1 and Q_5 are coprime [19].

In summary, as we pile up chiral primary particles on AdS_3 we get to a point at $J_L = J_R = k/2$ where we are on the verge of making a black hole [9]. If J_L, J_R approach their maximal values we have again a smooth geometry with a small number (if $J_{L,R}$ are sufficiently close to k) of chiral primary particles.

It would be interesting to see if something similar happens for other AdS spaces.

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